DOANE & INGALLS

Harmonics in the

Exciting Current of Transformers

Electrical Engineering

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HARMONICS IN THE EXCITING CURRENT OF TRANSFORMERS

BY

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AND
ROSS DARWIN INGALLS

THESIS

FOR THE

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

HARRY ALLAN DOANE AND ROSS DARWIN INGALLS

ENTITLED HARMONICS IN THE EXCITING CURRENT OF TRANSFORMERS

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF BACHELOR OF SCIENCE IN ELECTRICAL ENGINEERING

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HARMONICS IN THE EXCITING CURRENTS OF TRANSFORMERS .

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HARMONICS IN THE EXCITING CURRENTS OF TRANSFORMERS.

INTRODUCTION.

It is the purpose of this paper to study the harmonics in the exciting currents of transformers.Since the two lobes of an alternating wave are symmetrical with respect to the axis,only odd harmonics can exist because even harmonics would not produce symmetry.In the case of voltages,wave shapes due to harmonics have a great bearing on the life of insulation,especially in transformers working at high voltages.In such cases,with a peaked wave of electromotive force,the maximum value being large would cause a great injury to the insulation and thus have an important bearing on the operating conditions and design of transformers.

In order to study the harmonics appearing in the exciting currents,three core type transformers rated at I.5 K.W.,I200/2400 volts primary and I20/240 volts secondary,frequency 60 cycles,were used.Different three phase connections were made with a one to one ratio obtained by using the two secondaries of each transformer. Tests were also run on the transformers connected single phase.The alternator supplying power was a three phase,four pole,sixty cycle,220 volt machine which gave practically a sine wave of electromotive force.

We wish to express our warm appreciation and thanks to Mr.F.G.Willson whose advice and cooperation in bringing out the work materially assisted the writers.

HARMONICS IN THE EXCITING CURRENTS OF TRANSFORMERS.

GENERAL THEORY.

It has long been known that if a sine wave of electromotive force is impressed on an iron - clad circuit, such as a reactance coil or the primary coil of a transformer with the secondary open , that the current differs from a sine wave The electromotive force being a sine wave , the flux wave must be a sine wave for $e = nd\phi/dt$ and the current must take such a shape as will give this sine wave of flux. The current wave does not follow the sine law due to the fact that the hysteresis cycle is not symmetrical with respect to the coordinate axes but bends as the saturation point of the iron is approached.The irregular current wave can be broken up into sine waves called the fundamental , or the largest sine wave possible and smaller waves of higher frequencies which are called harmonics.

The shape of a wave depends upon the relative position of the fundamental and the harmonics and it can be proved that the core loss depends on the shape of the electromotive force wave and thus on the shape of the flux wave.Assuming two equal sine waves , each with a triple frequency wave superimposed as shown in the figuers below,the resultant wave in each case is determined by the addition of the fundamental sine wave and the triple frequency wave and the shapes of the resultant waves are greatly different from each other.

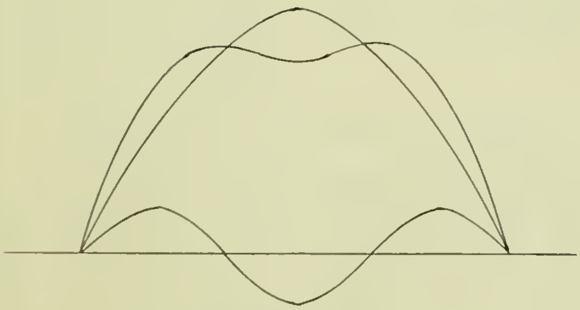

FIGURE I.

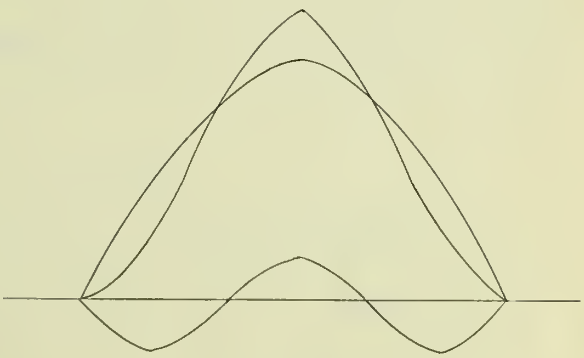

FIGURE 2

Let W = the hysteresis loss.

f = frequency.

V = volume of the iron.

B = flux density.

k = constant.

W_I = hysteresis loss for wave shape of Figure I.

W_2 = hysteresis loss for wave shape of Figure 2.

B_I = flux density for wave shape of Figure I.

B_2 = flux density for wave shape of Figure 2.

Then

$$\frac{W_1}{W_2} = \frac{B_1^{1.6}}{B_2^{1.6}}$$

for the hysteresis loss depends on the value of the volume of the iron and also the flux density raised to the one and six - tenths power, all multiplied by a constant.

The area under a wave divided by the length is equal to the average height, or if an electromotive force wave is used, the average value of electromotive force is obtained. It is known that,

$$e = n\frac{d\phi}{dt}$$

or by transposition $\quad edt = nd\phi.$

Then for a half cycle of the wave the integral between the limits zero and π of edt is equal to $n\Phi$, where Φ is the maximum value of the flux. The maximum value of the flux density multiplied by the cross - sectional area of the iron is equal to Φ or BA. If this value of BA is substituted in the integral $I/\pi\int_0^\pi edt = n\Phi/\pi$, the integral becomes equal to nAB/π or the average value of the electromotive force is equal to the integral.

Let E = the effective value of the two waves.

e_I = the average value of electromotive force for Figure I.

e_2 = the average value of electromotive force for Figure 2.

F_I = the form factor for Figure I.

F_2 = the form factor for Figure 2.

The form factor is defined as the effective value of the wave divided by the average value or ,

$$F = \frac{E}{e}$$

and by transposition

$$e = \frac{E}{F} .$$

Then

$$\frac{W_1}{W_2} = \frac{B_1^{1.6}}{B_2^{1.6}} = \frac{e_1^{1.6}}{e_2^{1.6}} = \left[\frac{E/F_1}{E/F_2}\right]^{1.6} = \left[\frac{F_2}{F_1}\right]^{1.6}.$$

This shows that the hysteresis loss depends on the shape of the wave , for the average value of Figure I is greater than that for Figure 2 and therefore the form factor for Figure I is less than that for Figure 2 and by inspection of the formula , the hysteresis loss for Figure I is greater than that for Figure 2. The shape of the flux wave depends on the shape of the electromotive force wave and as already explained the current wave must take such a shape as will give a sine wave of flux if the electromotive force is a sine wave.And from this it is seen that the shape of the current wave depends on the hysteresis cycle.

The exciting current wave can be derived step by step from the hysteresis loop and its shape depends upon the bend of the magnetic characteristic and is not due to the energy loss or the area of the loop.The wave of current due to excitation can be resolved into two component waves , one an energy component i' in phase with the electromotive force and a wattless component in quadrature with it.For example in Figure 3 , E is a sine wave of electromotive force , B is a sine wave of flux and I is the exciting current wave.

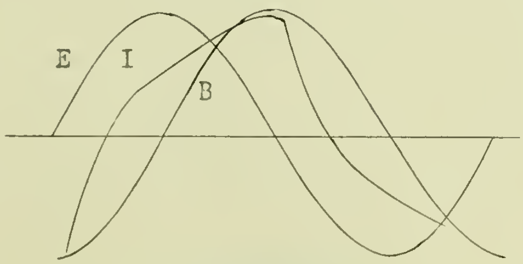

FIGURE 3.

As said before the exciting current can be broken up into wattless and energy components.These components , the hysteresis power current i' , and the reactive magnetizing current i'' are plotted in Figure 4 and show that i' is nearly a sine wave and i'' is greatly distorted . Thus the wattless or magnetizing current component of the exciting current causes the distortion .

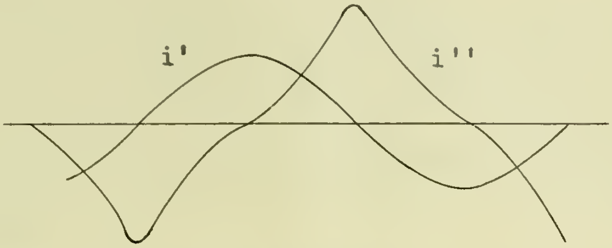

FIGURE 4.

SINGLE PHASE .

The best method of studying the exciting current is by means of the oscillo-graph . In the investigation of the harmonics causing distorted waves it is necessary to know the shape of the electromotive force wave impressed upon the transformer circuits or primaries .

Plate I shows the reproduction of the oscillogram taken of the electromotive force wave of a three phase alternator used to supply power in the tests . Upon examination of this wave , it is seen that it is very nearly a sine wave , only high harmonics existing , the magnitude of which were too small to cause any appreciable error when neglected so that the wave may be considered a sine wave . It is explained in the preceeding paragraphs that the flux wave will be a sine wave and that the exciting current will have a distorted appearance .

Plate 2 shows the exciting current wave for a single transformer with the electromotive force shown in Plate I impressed . This wave is distorted and peaked and was worked at a density of 118,000 lines per square inch . It shows the fundamental , triple and fifth harmonics . Plate 3 shows the wave taken under similar conditions but at a lower density of 85,000 lines per square inch . It is noticed that the wave is different from that of Plate 2 in that the magnitudes of the harmonics are different and that the higher the density the greater is the distortion due to a greater magnetizing current . The harmonics as the density is increased keep moving more and more in phase with the fundamental and give a more peaked wave.

PLATE NO. I.

ELECTROMOTIVE FORCE WAVE OF MACHINE.

PLATE NO.2.

EXCITING CURRENT AND HARMONICS IN A SINGLE TRANSFORMER.

DENSITY   II8,000 LINES PER SQUARE INCH.

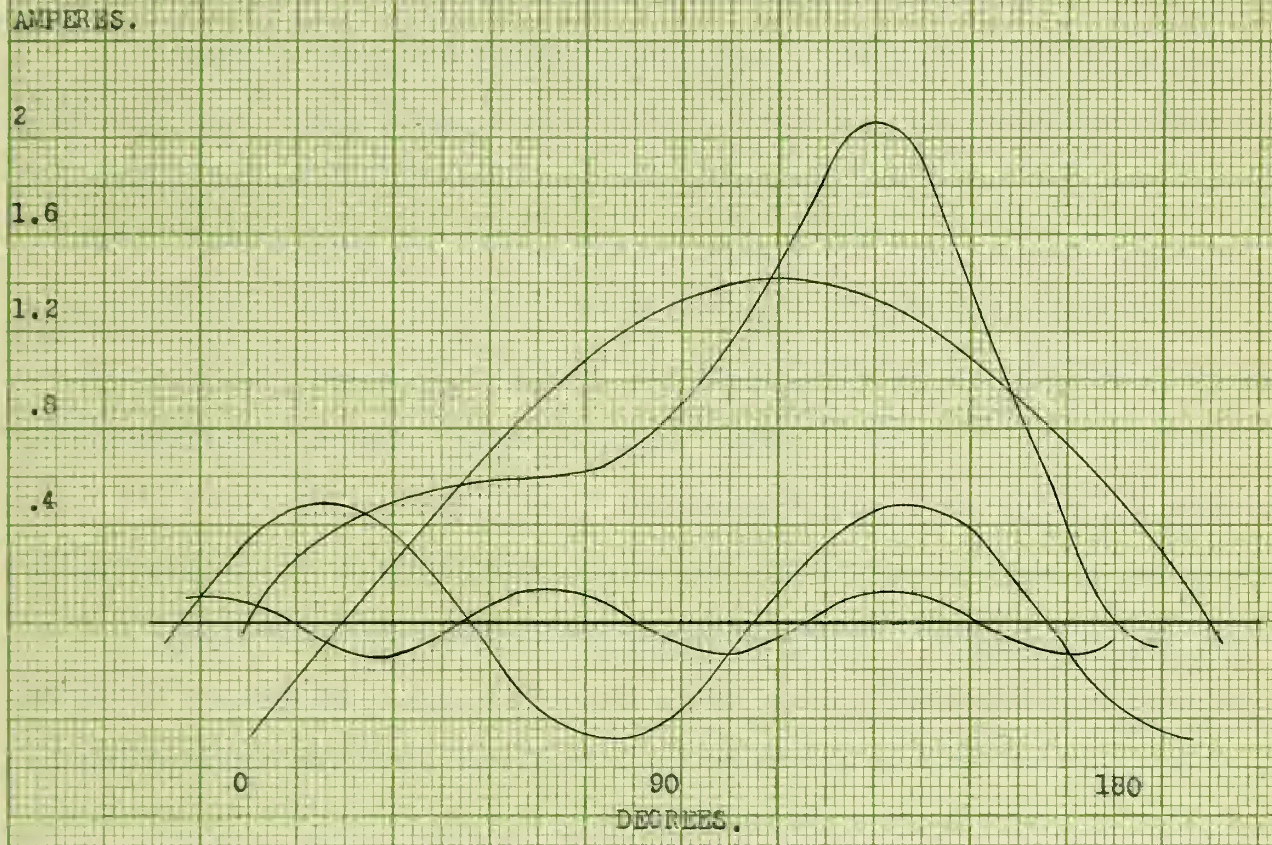

PLATE NO.3.

EXCITING CURRENT AND HARMONICS IN A SINGLE TRANSFORMER.

DENSITY 85000 LINES PER SQUARE INCH.

THREE PHASE CONNECTIONS.

In three phase transformer connections , the wave shape distortions are important factors.The currents in a three wire,three phase system are one-hundred and twenty degrees out of phase with each other and the sum of the three currents is zero , provided the system is balanced or when all three transformers take the same exciting current . The third harmonics are displaced from each other by three times one-hundred and twenty degrees , or in other words are in phase with each other.Therefore the third harmonics cannot flow.

In a three phase system of transformers with their primaries connected in delta there can be no third harmonics in the line currents . This is due to the fact that the three currents in the lines add up to zero and if there were any third harmonics the sum would not be zero since the third harmonics are in phase.Hence it is self evident that there can be no third harmonics in the line currents.In a system similar to the one above but with the primaries connected in star there can be no third harmonics in the line currents for the three currents add up to zero and if there were triple harmonics the three currents in the lines could not add up to zero.

Refering to Plates 4 and 5 the triple is very small.There would have been no triple if the transformers had been perfectly balanced.Plate 4 shows the line current wave with star connected primary and delta secondary.Plate 5 shows the line current wave with delta connected primary and open delta secondary.The wave distortion differs greatly in the two cases , due to the fact that the fifth harmonic in both cases is large and has a different angle of phase displacement relative to the fundamental . Higher harmonics other than the third and its multiples were negligable.

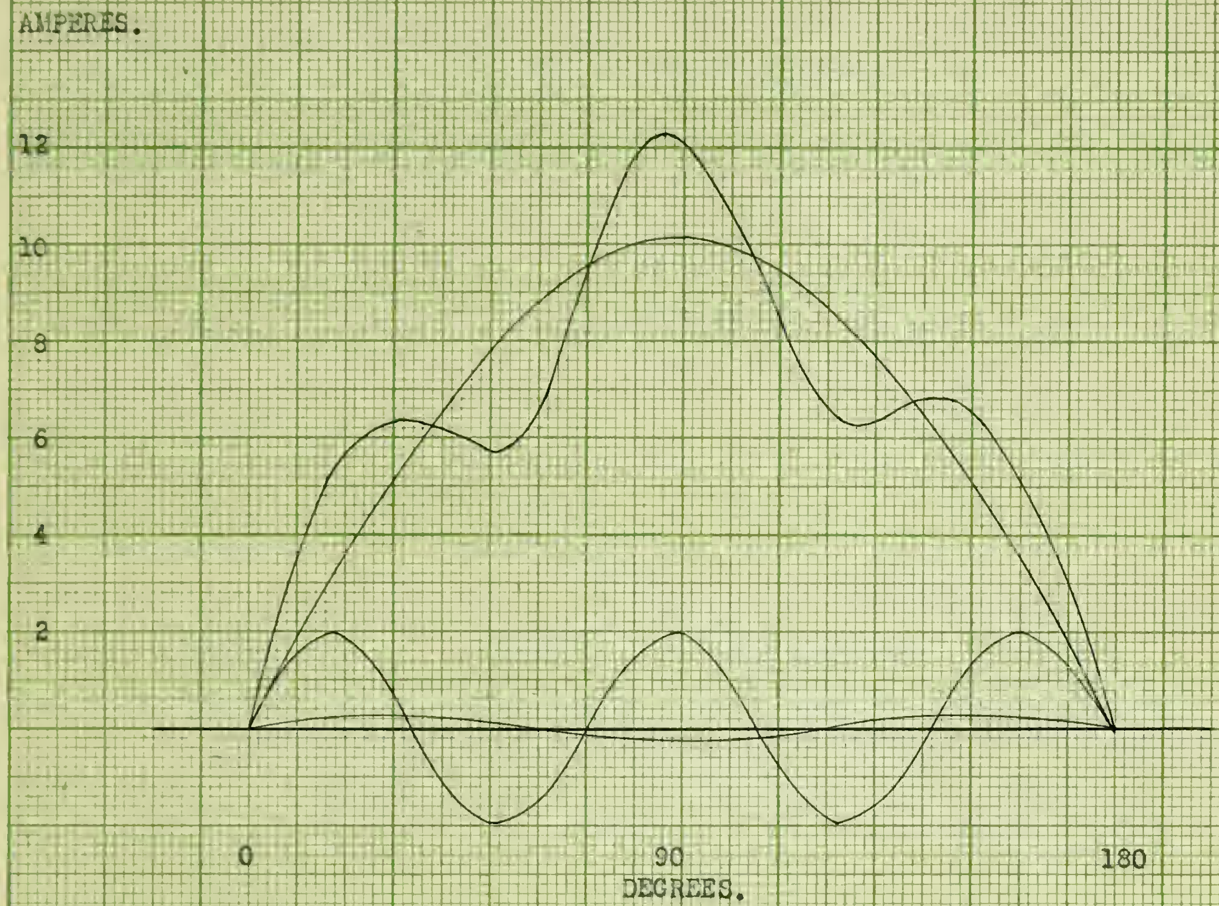

PLATE NO.4.

LINE CURRENT AND HARMONICS.

STAR PRIMARY - CLOSED DELTA SECONDARY.

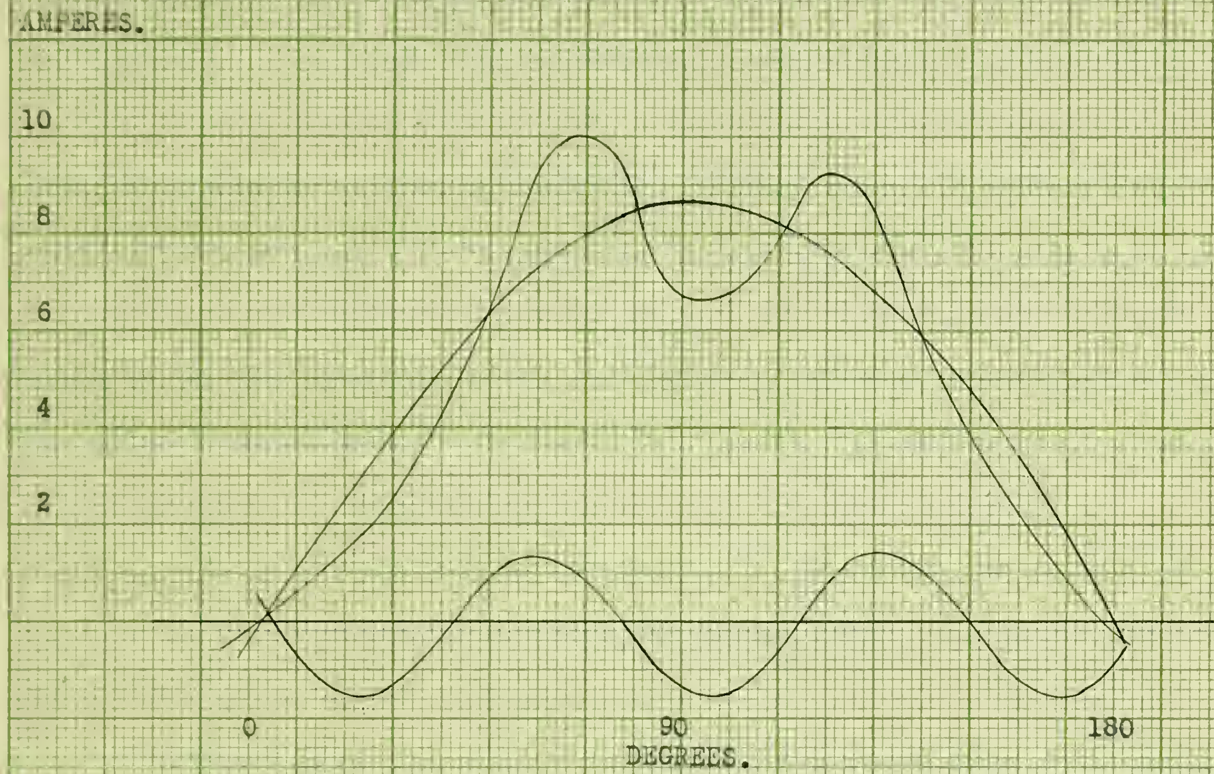

PLATE NO.5.

LINE CURRENT AND HARMONICS.

DELTA PRIMARY - OPEN DELTA SECONDARY.

With delta connected primaries there can be no triple in the line currents as already shown , but triple harmonics can exist in the local closed circuit . With star connected primaries the triple harmonics cannot exist in the lines or in the coils as is self - evident from the study of the star connections from which it is seen that the coil current and the line current are the same . Plate 6 shows the primary coil current with star connections in the primary and open delta secondary . As shown by the analysis of this wave a large fifth harmonic exists which causes the wave of current to be greatly distorted . Plate 7 shows the primary coil current for delta primary and open delta secondary and this reproduction shows great wave distortion due to the third and fifth harmonics . The distortion of the waves are different in the two cases due to the difference of angle of phase displacement of the harmonics relative to their fundamentals . In the case of the delta primary , the triple harmonic has a greater distortion than the other harmonics and it shows that with this connection the primary current has a greater peak or maximum value than in the case of the star connection where the fifth harmonic causes a double peak rather than a single peak due to a triple .

Plate 8 gives the wave of the circulating triple when the primaries were connected in star and the secondaries in open delta . The triple occurs because the third harmonics in the three transformer secondaries are in series and the fundamentals and higher harmonics blot themselves out . Mathematically this can be proved in the following manner . let e_1 , e_2 and e_3 represent instantaneous values of electromotive force in the respective coils and let E_1 be the maximum value of the fundamental and E_3 be the maximum value of the third harmonic . Let θ be the angle of phase displacement . Then

$$e_1 = E_1 \sin \theta + E_3 \sin 3\theta \quad .....$$

$$e_2 = E_1 \sin (\theta - 120) + E_3 \sin (3\theta - 360)$$

$$e_3 = E_1 \sin (\theta - 240) + E_3 \sin (3\theta - 720)$$

$$e_1 + e_2 + e_3 = E_1 \sin \theta - E_3 \sin 3\theta$$

$$- 1/2\, E_1 \sin \theta - \sqrt{3}/2\, E_1 \cos \theta + E_3 \sin 3\theta$$

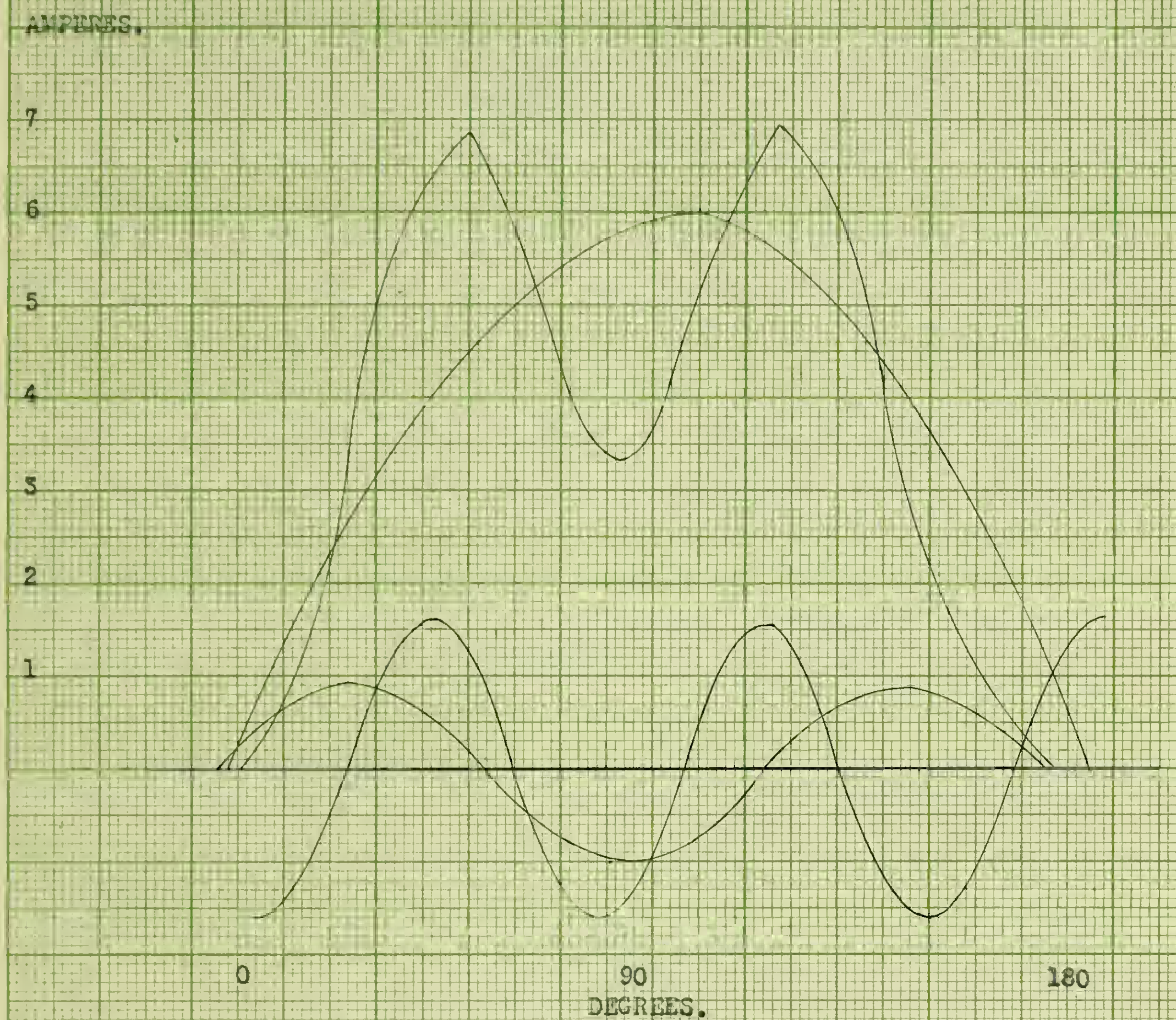

PLATE NO.6.

PRIMARY COIL CURRENT AND HARMONICS.

STAR PRIMARY - OPEN DELTA SECONDARY.

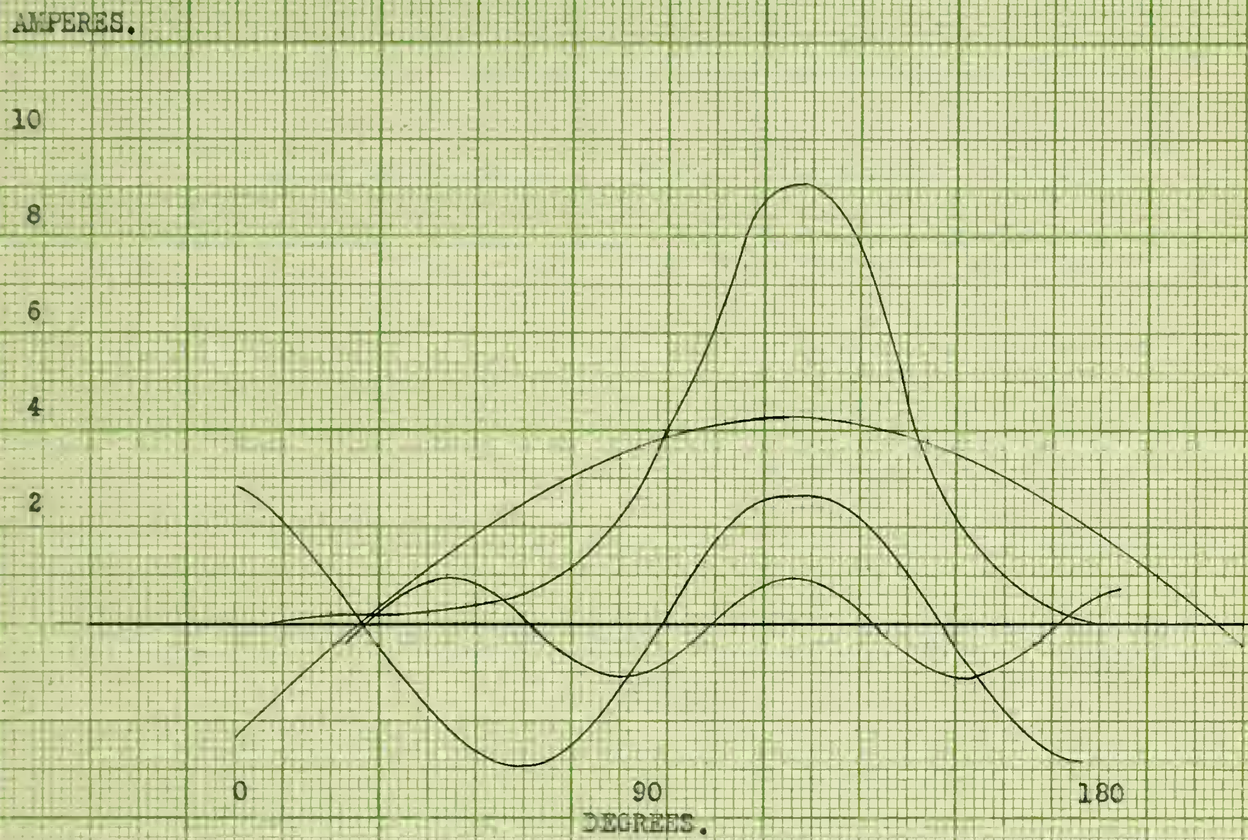

PLATE NO.7.

PRIMARY COIL CURRENT AND HARMONICS.

DELTA PRIMARY - OPEN DELTA SECONDARY.

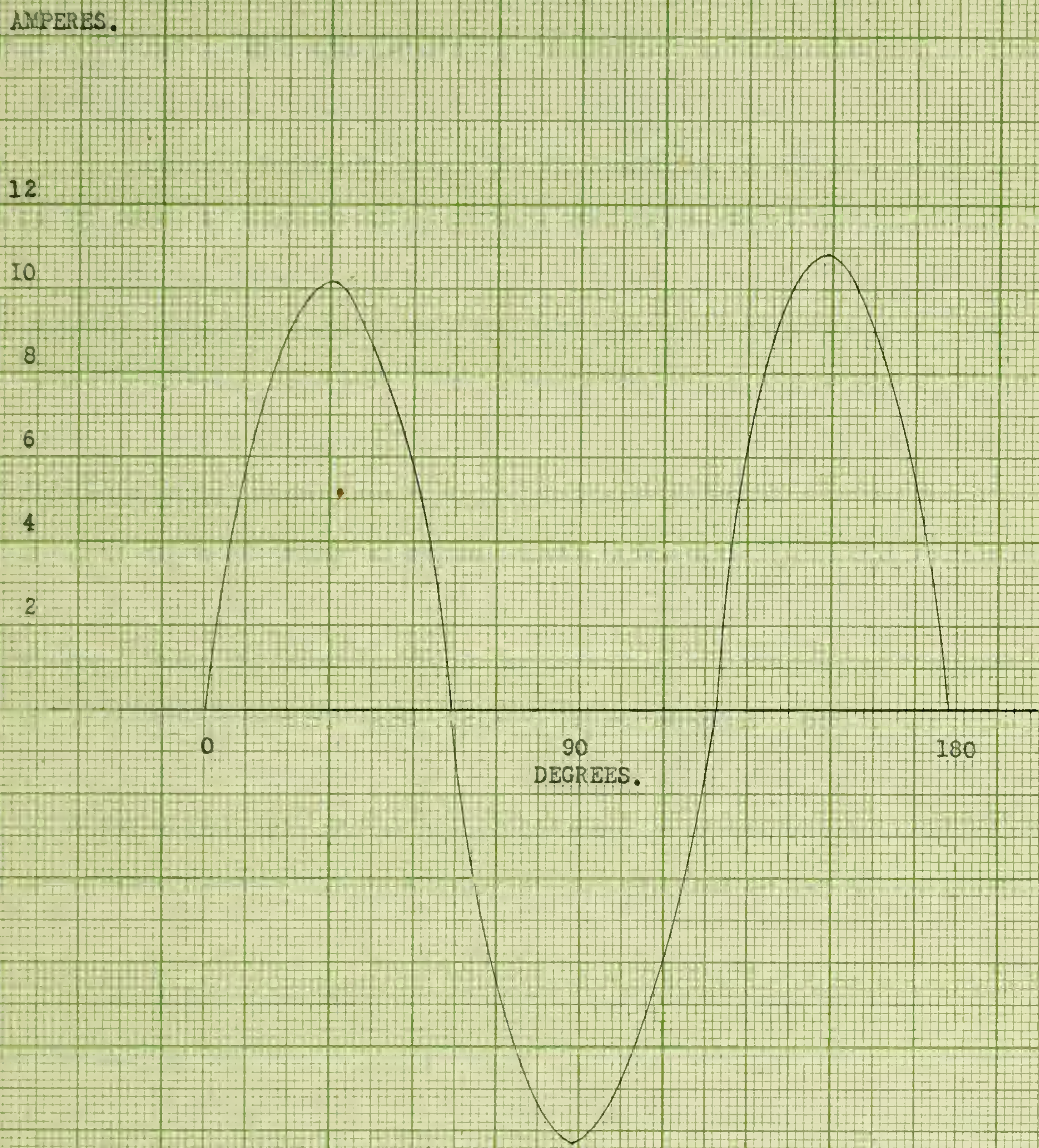

PLATE NO.8.

CIRCULATING TRIPLE CURRENT.

STAR PRIMARY - OPEN DELTA SECONDARY.

$$- 1/2\, E_1 \sin\theta + \sqrt{3}/2\, E_1 \cos\theta + E_3 \sin 3\theta\ .$$

Thus it is seen by cancelling like terms with opposite signs ,

$$e_1 + e_2 + e_3 = 3E_3 \sin 3\theta\ ,$$

which shows that nothing but the third harmonic can exist .

Plate 9 shows the triple in the secondary for delta primary and delta secondary connection . It will be noticed that the maximum for this wave is less than the maximum in Plate 8 . This difference is due to the fact that in the conditions shown in Plate 8 , there is only one path for the triple harmonic , that is the path in the delta secondary as the primary was connected in star . In the conditions shown in Plate 9 there are two paths , one in the primary and one in the secondary for circulating currents as both are connected in delta .

Plate 10 shows the primary coil current wave for a star connected primary and a triple frequency wave for the delta connected secondary with the same star primary . From the previous study the position of the harmonics has much to do with the shape of the current wave . The effective value of the wave remains the same no matter what the shape the wave is . Mathematically the effective value can be proved to be the square root of the sum of the squares of the effective values of the harmonics and fundamental as follows . Let the equation of an alternating current wave be

$$i = I_1 \sin\phi + I_3 \sin 3\phi + \ \ldots\ I_1 \cos\phi + I_3 \cos 3\phi + I_5 \cos 5\phi + \ \ldots$$

squaring both sides ,

$$i^2 = I_1^2 \sin^2\phi + I_3^2 \sin^2 3\phi + \ \ldots\ I_1^2 \cos^2\phi + I_3^2 \cos^2 3\phi + I_5^2 \cos^2 5\phi + \ \ldots\ I_1 \sin\phi I_1 \cos\phi +$$
$$I_3 I_1 \cos\phi \sin 3\phi + I_1 I_3 \cos 3\phi \sin\phi + \ldots$$

multiplying both sides by $d\phi$,

$$i^2 d\phi = I_1^2 \sin^2\phi\, d\phi + I_3^2 \sin^2 3\phi\, d\phi + I_5^2 \sin^2 5\phi\, d\phi + \ldots\ I_1^2 \cos^2\phi\, d\phi + I_3^2 \cos^2 3\phi\, d\phi + I_5^2 \cos^2 5\phi\, d\phi + \ \ldots$$
$$I_1 I_3 \cos\phi \sin\phi\, d\phi + I_3 I_1 \cos\phi\ \sin 3\phi\, d\phi + \ldots$$

taking the integral of both sides between limits zero and 2π , to get the area of the curve ,

PLATE NO.9.

SECONDARY CURRENT.

PRIMARY DELTA - SECONDARY CLOSED DELTA.

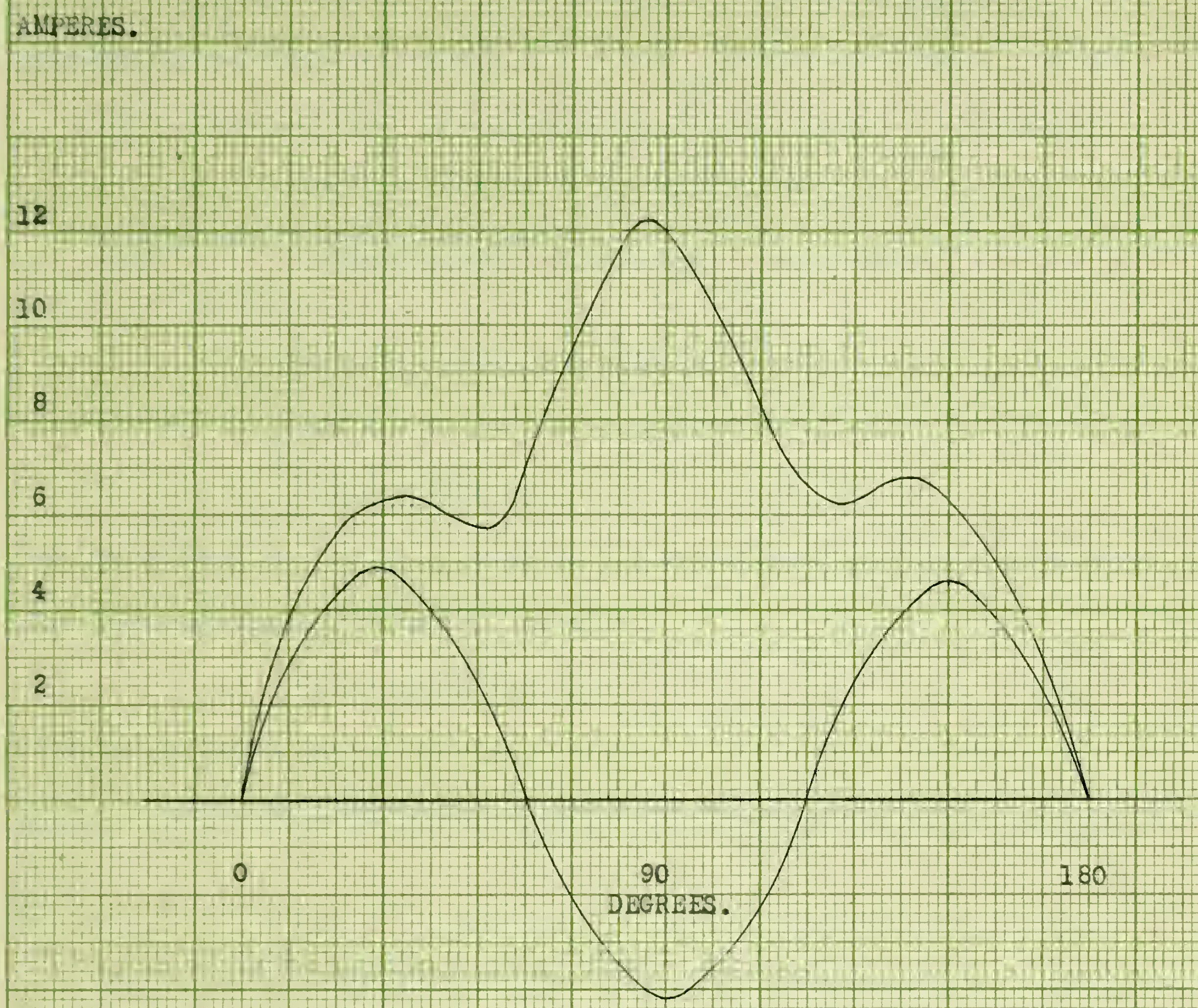

PLATE NO. 10.

PRIMARY COIL CURRENT AND SECONDARY TRIPLE.

STAR PRIMARY - DELTA SECONDARY.

$$\int_0^{2\pi} i^2 d\phi = I_1^2\int_0^{2\pi}\sin^2 d\phi + I_3^2\int_0^{2\pi}\sin^2 3\phi d\phi + I_5^2\int_0^{2\pi}\sin^2 5\phi d\phi + \ldots .\; I_1^2\int_0^{2\pi}\cos^2 d\phi - I_3^2\int_0^{2\pi}\cos^2 3\phi d\phi +$$
$$I_5^2\int_0^{2\pi}\cos^2 5\phi d\phi + \ldots .\; I_1^2\int_0^{2\pi}\sin\phi\cos\phi d\phi + I_1 I_3\int_0^{2\pi}\cos\phi\sin 3\phi d\phi + \ldots .$$

integrating each one separately ,

$$I_1^2\int_0^{2\pi}\sin^2\phi d\phi = I_1^2\int_0^{2\pi}(\ 1/2 - \cos 2\phi/2\ )d\phi = I_1^2\int_0^{2\pi}(\ 1/2\ \ d\phi - 1/2\int_0^{2\pi}\cos 2\phi d\phi =$$
$$I_1^2\,\phi/2\Big|_0^{2\pi} + (\ I_1^2/2\ \sin 2\phi\ )\Big|_0^{2\pi} = 2I_1^2\pi/2 = I_1^2\pi.$$
$$I_3^2\int_0^{2\pi}\sin\ 3\phi d\phi = 3I_3^2/3\int_0^{2\pi}\sin^2\phi d\phi = I_3^2\pi.$$

Formula of integration

$$\int\sin^2 x dx = x/2 - 1/2\ \sin x\cos x$$
$$\int\cos^2 x dx = x/2 + 1/2\ \sin x\cos x$$
$$\int I_1^2\cos\phi\sin\phi d\phi = 0\ .$$

For the product of two sine waves is a sine wave and the integral between zero and 2π is zero .Therefore the total area is ,

$$I_1^2\pi + I_3^2\pi + I_5^2\pi + \ \ldots .\ldots\ I_1^2\pi + I_3^2\pi + I_5^2\pi + \ldots .\ldots$$

Then the ordinate is ,

$$\frac{I_1^2\pi + I_3^2\pi + I_5^2\pi + \ \ldots .\ldots\ I_1^2\pi + I_3^2\pi + I_5^2\pi + \ldots .\ldots}{2\pi}$$

or

$$\frac{I_1^2 + I_3^2 + I_5^2 + \ldots .\ldots\ I_1^2 + I_3^2 + I_5^2 + \ldots .\ldots}{2}$$

or

$$i(eff) = \sqrt{\frac{I_1^2 + I_3^2 + I_5^2 + \ldots .\ldots\ I_1^2 + I_3^2 + I_5^2 + \ldots .\ldots}{2}}$$

which is the proof required .

The same results can be obtained graphically by reversing the triple harmonic in Plate 10 . This can be done because the triple frequency harmonic is 180 degrees out of phase with the primary current . The resultant wave shown in Plate 10a has the same effective value as the original wave but the distortions are different and the maximum value of 10a is the greater .

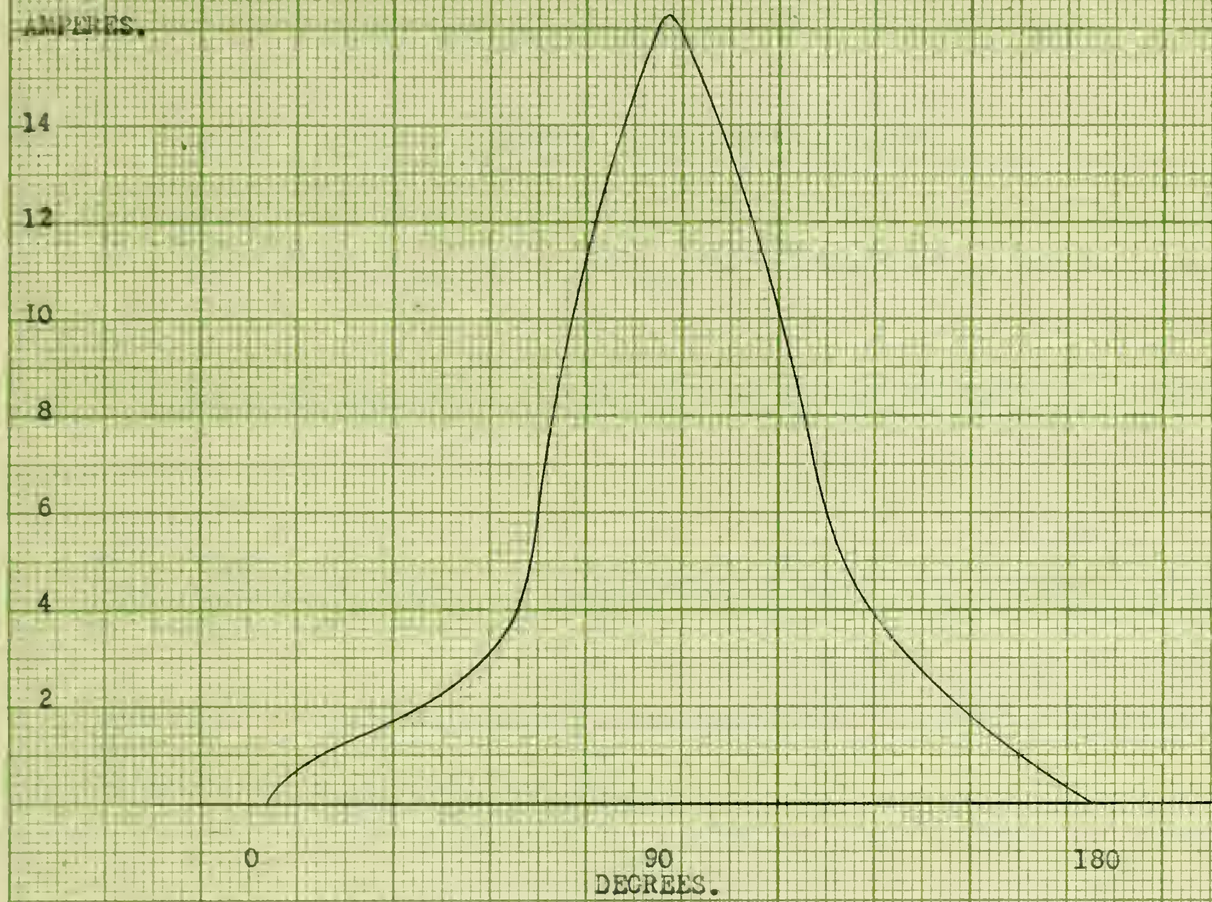

PLATE NO.10a.

WAVE OBTAINED BY ADDING THOSE

SHOWN IN PLATE NO.10.

FORMULA USED IN WAVE ANALYSIS .

Any univalent periodic function may be expressed by Fourier's series as follows ,

$$y = a_0 + a_1\cos\theta + a_2\cos 2\theta + \dots\ a_n\cos n\theta + b_1\sin\theta + b_2\sin 2\theta + b_3\sin 3\theta + \dots\ b_n\sin n\theta\ .$$

To determine the constants a_0 , a_1 , etc. , multiply thru by $d\theta$ and integrate over 360 degrees .

$$\int_0^{2\pi} y\,d\theta = a_0\int_0^{2\pi} d\theta + a_1\int_0^{2\pi}\cos\theta\,d\theta + \dots\ a_n\int_0^{2\pi}\cos n\theta\,d\theta + b_1\int_0^{2\pi}\sin\theta\,d\theta + b_2\int_0^{2\pi}\sin 2\theta\,d\theta + \dots\ b_n\int_0^{2\pi}\sin n\theta\,d\theta$$

or

$$\int_0^{2\pi} y\,d\theta = a_0\,\theta\Big|_0^{2\pi} + a_1\,\sin\theta\Big|_0^{2\pi} + a_2\,\sin 2\theta/2\Big|_0^{2\pi} + \dots\ a_n\sin n\theta/n\Big|_0^{2\pi} - b_1\cos\theta\Big|_0^{2\pi} - b_2\cos 2\theta/2\Big|_0^{2\pi} + \dots$$

All terms vanish except the first and ,

$$\int_0^{2\pi} y\,d\theta = 2\pi a_0$$

$$a_0 = I/2\pi\int_0^{2\pi} y\,d\theta$$

But $\int y\,d\theta$ is the area of the curve over the space of 360 degrees .

Let

$$A = \int_0^{2\pi} y\,d\theta$$

then

$$a_0 = A\ /2\pi.$$

But $A\ /\ 2\pi$ is the mean ordinate of the wave . Hence a may be found by taking the numerical mean of a sufficient number of ordinates over 360 degrees .

To find any other value as a multiply the equation thru by $\cos n\theta\,d\theta$ and integrate over 360 degrees .

$$\int_0^{2\pi} y\cos n\theta\,d\theta = a_0\int_0^{2\pi}\cos n\theta\,d\theta + a_1\int_0^{2\pi}\cos\theta\cos n\theta\,d\theta + a_2\int_0^{2\pi}\cos 2\theta\cos n\theta\,d\theta + \dots\ a_n\int_0^{2\pi}\cos^2 n\theta\,d\theta + \dots$$
$$b_1\int_0^{2\pi}\sin\theta\cos n\theta\,d\theta + b_2\int_0^{2\pi}\sin 2\theta\cos n\theta\,d\theta + \dots\ b_n\int_0^{2\pi}\sin n\theta\cos n\theta\,d\theta\ .$$

or

$$\int_0^{2\pi} y\cos n\theta\,d\theta = a_0\int_0^{2\pi}\cos n\theta\,d\theta + a_1\int_0^{2\pi} I/2\left[\cos\,(n+I)\theta + \cos(n-I)\theta\right]d\theta +$$

$$a_2\int_0^{2\pi} I/2\left[\cos 2(n+2)\theta + \cos(n-2)\theta\right] d\theta + \ldots\ a_n\int_0^{2\pi} I/2(I+\cos 2n\theta)d\theta +$$
$$b_1\int_0^{2\pi} I/2\ \left[\sin(n+I)\theta - \sin(n-I)\theta\right] d\theta + b_2\int_0^{2\pi} I/2\left[\sin(n+2)\theta - \sin(n-2)\theta\right] d\theta$$
$$+\ldots.\ b_n\int_0^{2\pi} I/2\sin 2n\theta d\theta + \ldots.$$

All these integrals vanish except

$$\int_0^{2\pi} y\cos n\theta d\theta = a_n\int_0^{2\pi} I/2 d\theta = a_n(\theta/2) = a_n\pi$$

which is the area of the new derived wave .

Hence

$$a_n = I/\pi\int_0^{2\pi} y\cos n\theta d\theta\ .$$

The integral may be interpreted as follows ; if the instantaneous values of y in the original curve be multiplied by $\cos n\theta$ and a wave plotted from these values the integral $y\cos n\theta d\theta$ between the limits zero and 2π is the area of this derived curve. Call this as found A_N, then the average ordinate is $A_N/2\pi$. Hence $a_n = 2(A_N/2\pi)$ or twice the average of the numbers found by multiplying the instantaneous values of y by corresponding values of $\cos n\theta$.

The value of b_n is found in a similar way by multiplying by $\sin n\theta$ or

$$\int_0^{2\pi} y\sin n\theta d\theta = a_0\int_0^{2\pi}\sin n\theta d\theta + a_1\int_0^{2\pi}\cos\theta\sin n\theta d\theta + a_2\int_0^{2\pi}\cos 2\theta\sin n\theta d\theta + \ldots\ a_n\int_0^{2\pi}\cos n\theta\sin n\theta d\theta +$$
$$b_1\int_0^{2\pi}\sin\theta\sin n\theta d\theta + b_2\int_0^{2\pi}\sin 2\theta\sin n\theta d\theta + \ldots.\ b_n\int_0^{2\pi}\sin^2 n\theta d\theta = a_0\int_0^{2\pi}\sin n\theta d\theta +$$
$$a_1\int_0^{2\pi} I/2\left[\sin(n+I)\theta + \sin(n-I)\theta\right] d\theta + a_2\int_0^{2\pi} I/2\left[\sin(n+2)\theta + \sin(n-2)\theta\right] d\theta +$$
$$\ldots.\ a_n\int_0^{2\pi} I/2\sin an\theta d\theta + b_1\int_0^{2\pi} I/2\left[\cos(n-I)\theta - \cos(n+I)\theta\right] d\theta +$$
$$b_2\int_0^{2\pi} I/2\left[\cos(n-2)\theta - \cos(n+2)\theta\right] d\theta + \ldots\ b_n\int_0^{2\pi} I/2(I - \cos an\theta)d\theta + \ldots. =$$
$$b_n\int_0^{2\pi} I/a d\theta = \pi b_n. \text{ Hence}$$
$$b_n = I/\pi\int_0^{2\pi} y\sin n\theta d\theta\ .$$

As before the integral is a curve derived by multiplying instantaneous ordinates of y by corresponding values of $\sin n\theta$. Calling this value A'_N, $b_n = 2(A'_N/2\pi)$, or twice the average of the numerical values obtained for the derived curve .

It is seen that the value of any required harmonic may be solved for in this manner . When even harmonics are not present and the wave is symmetrical about its axis then the average may be taken over a half wave or from zero to π. The wave is then expressed in the following form ,

$y = a_1 \cos\theta + a_3 \cos 3\theta + a_5 \cos 5\theta + \ldots\ b_1 \sin\theta + b_3 \sin 3\theta + b_5 \sin 5\theta + \ldots.$

The odd and even harmonics may be separated as follows : if to the numerical values of the ordinates from zero to 180 degrees are added those for corresponding angles from 180 to 360 degrees , the resultant wave will contain only even harmonics . If those from 180 to 360 degrees are subtracted from those from zero to 180 degrees the resultant wave will contain only odd harmonics . And as seen before only odd harmonics exist in alternating current waves .

Having any harmonic of the form of the binominal , $a_n \cos n\theta + b_n \sin n\theta$, it may be transformed as follows . Let $\tan \phi_n = b_n/a_n$ and let $c_n = \sqrt{a_n^2 + b_n^2}$. Then

$\cos \phi_n = \dfrac{a_n}{\sqrt{a_n^2 + b_n^2}}$ and $a_n = \cos\phi_n \sqrt{a_n^2 + b_n^2}$ and $b_n = \sin\phi_n \sqrt{a_n^2 + b_n^2}$.

Substituting these values in the binominal $a_n \cos n\theta + b_n \sin n\theta = c_n \cos(n\theta - \phi_n)$. From these values of a_n and b_n the value of ϕ_n and its sign may be found . It must be remembered in evaluation , that $\sin \phi_n$ is positive in the first and second quad - rants and negative in the third and fourth ; $\cos \phi_n$ is negative in second and third quadrants and positive in the first and fourth or when ,

$\tan \phi_n = \dfrac{b_n}{a_n}$ ϕ_n is between 0 and 90 degrees .

$\tan \phi_n = \dfrac{b_n}{-a_n}$ ϕ_n is between 90 and 180 degrees .

$\tan \phi = \dfrac{-b_n}{-a_n}$ ϕ_n is between 180 and 270 degrees .

$\tan \phi = \dfrac{-b_n}{a_n}$ ϕ_n is between 270 and 360 degrees .

From this solution the final equation will contain cosine functions . These may be transformed into sine functions by remembering that $\cos \phi_n = -\sin(\phi_n - 90)$. For some purposes one form is more convenient than the other . The final equation will appear in the form

$$y = a_0 + c_1 \cos(\theta - \phi_1) + c_2 \cos(2\theta - \phi_2) + \ldots\ c_n \cos n(\theta - \phi_n) \text{ or}$$

$$= a_0 + c_1 \cos(\theta - \phi_1) + c_2 \cos 2(\theta - \phi_2/2) + \ldots\ c_n \cos n(\theta - \phi_n/n) .$$

The following is an illustration of wave analysis of one of the oscillograms taken of the primary coil current of star connected primary and delta secondary . The first two columns of each harmonic and the fundamental are the abscissa in degrees and the ordinates in amperes respectively . The last column gives the ordinate

corresponding to the abscissa for the fundamental or the harmonic . Pages 26 , 27 , and 28 show this data and the wave and derived harmonics are shown on Page 29 .

WAVE ANALYSIS DATA .

FUNDAMENTAL .

θ	i	$\cos\theta$	$i\cos\theta$	$\sin\theta$	$i\sin\theta$	$(\theta - \phi_I)$	$\cos(\theta - \phi_I)$	$c_I\cos(\theta - \phi_I)$
0	0	I	0	0	0000	-89.5	.009	.09
I0	3.7	.984	3.64	.173	.64	-79.5	.I82	I.835
20	5.6	.939	5.25	.342	I.92	-69.5	.35	3.53
30	6.3	.866	5.46	.5	3.I5	-59.5	.508	5.I2
40	6.2	.766	4.75	.643	3.99	-49.5	.649	6.55
50	5.7	.643	3.67	.766	4.37	-39.5	.772	7.79
60	6.5	.500	3.25	.866	5.63	-29.5	.87	8.78
70	9.4	.342	3.2I	.939	8.82	-I9.5	.943	9.52
80	II.7	.I73	2.03	.984	II.5I	- 9.5	.986	9.95
90	I2	0000	0000	I.000	I2	.5	I.000	I0.09
I00	I0.6	-.I73	-I.84	.984	I0.42	I0.5	.983	9.92
II0	8.4	-.342	-2.87	.939	7.89	20.5	.937	9.45
I20	6.7	-.500	-3.35	.866	5.8	30.5	.862	8.7
I30	6.3	-.643	-4.05	.766	4.83	40.5	.76	7.67
I40	6.7	-.766	-5.I3	.643	4.3I	50.5	.636	6.42
I50	6.5	-.866	-5.63	.5	3.25	60.5	.492	4.96
I60	5.2	-.939	-4.88	.342	I.78	70.5	.334	3.37
I70	3.0	-.984	-2.95	.173	.52	80.5	.I65	-I.655
I80	0	-I	0	0	0	90.5	-.009	- .09
			.66		90.82			

$a_I = .0733$, $b_I = I0.09I$, $c_I = I0.09I$, $\tan\phi_I = I37.5$, $\phi_I = 89.5$ degrees .

THIRD HARMONIC .

3θ	i	cos3θ	icos3θ	sin3θ	isin3θ	$3\theta - \phi_3$	cos$(3\theta - \phi_3)$	$c_3$cos$(3\theta - \phi_3)$
0	0	I	0	00	000	-8I.5	.I48	.04
30	3.7	.866	3.2	.5	I.85	-5I.5	.636	.I75
60	5.6	.5	2.8	.866	4.85	-2I.5	.93	.256
90	6.3	00	0	I	6.3	8.5	.989	.272
I20	6.2	-.5	-3.I	.866	5.35	38.5	.783	.2I5
I50	5.7	-.866	-4.93	.5	2.85	68.5	.367	.I0I
I80	6.5	-I	-6.5	00	000	98.5	-.I48	-.04
2I0	9.4	-.866	-8.I3	- .5	-4.7	I28.5	-.636	-.I75
240	II.7	-.5	-5.85	- .866	-I0.I3	I58.5	-.93	-.256
270	I2	000	000	-I.	-I2.00	I88.5	-.989	-.272
300	I0.6	.5	5.3	- .866	-9.I7	2I8.5	-.783	-.2I5
330	8.4	.866	7.27	- .5	-4.2	248.5	-.367	-.I0I
360	6.7	I	6.7	000	0000	278.5	.I48	.04
390	6.3	.866	5.45	.5	3.I5	308.5	.636	.I75
420	6.7	.5	3.35	.866	5.8	338.5	.93	.256
450	6.5	00	000	I.0	6.5	368.5	.989	.272
480	5.2	-.5	-2.6	.866	4.5	398.5	.783	.2I5
5I0	3.0	-.866	-2.59	.5	I.5	428.5	.367	.I0I
540	00	-I	0000	000	0000	458.5	-.I48	-.04
			.365		2.45			

$a_3 = .0405$, $b_3 = .272$, $c_3 = .275$, $\tan\phi_3 = 6.7I$, $\phi_3 = 8I.5$ degrees .

FIFTH HARMONIC .

5θ	i	$\cos 5\theta$	$i\cos 5\theta$	$\sin 5\theta$	$i\sin 5\theta$	$5\theta - \phi_5$	$\cos(5\theta - \phi_5)$	$c_5\cos(5\theta - \phi_5)$
0	0	I	0	0	0	-82	.I39	.284
50	3.7	.643	2.38	.766	2.835	-32	.848	1.735
I00	5.6	-.I73	- .97	.984	5.5I	I8	.951	1.945
I50	6.3	-.866	-5.46	.5	3.I5	68	.375	.767
200	6.2	-.939	-5.82	-.342	-2.I2	II8	-.47	-.96
250	5.7	-.342	-I.95	-.939	-5.35	I68	-.978	-.2
300	6.5	.5	3.25	-.866	-5.63	2I8	-.778	-1.59
350	9.4	.984	9.25	-.I73	-I.625	268	-.035	- .0717
400	II.7	.766	8.96	.642	7.53	3I8	.743	1.52
450	I2	0000	0000	I.0	I2.0	368	.99	2.04
500	I0.6	-.766	-8.I2	.643	6.82	4I8	.53	1.083
550	8.4	-.984	-8.26	-.I73	-I.45	468	-.309	- .631
600	6.7	-.5	-3.35	-.866	-5.8	5I8	-.927	-1.9
650	6.3	.342	2.I6	-.939	-5.92	568	-.883	-1.805
700	6.7	.939	6.29	-.342	-2.29	6I8	-.375	- .766
750	6.5	.866	5.63	.5	3.25	668	.616	1.26
800	5.2	.I73	.88	.984	5.02	7I8	.035	.0717
850	3.0	-.643	-I.93	.766	2.3	768	.669	1.368
900	000	- I.0	0000	0000	0000	8I8	-.139	- .284
			2.52		I8.23			

$a_5 = .28$, $b_5 = 2.025$, $c_5 = 2.045$, $\tan\phi_5 = 7.17$, $\phi_5 = 82$ degrees .

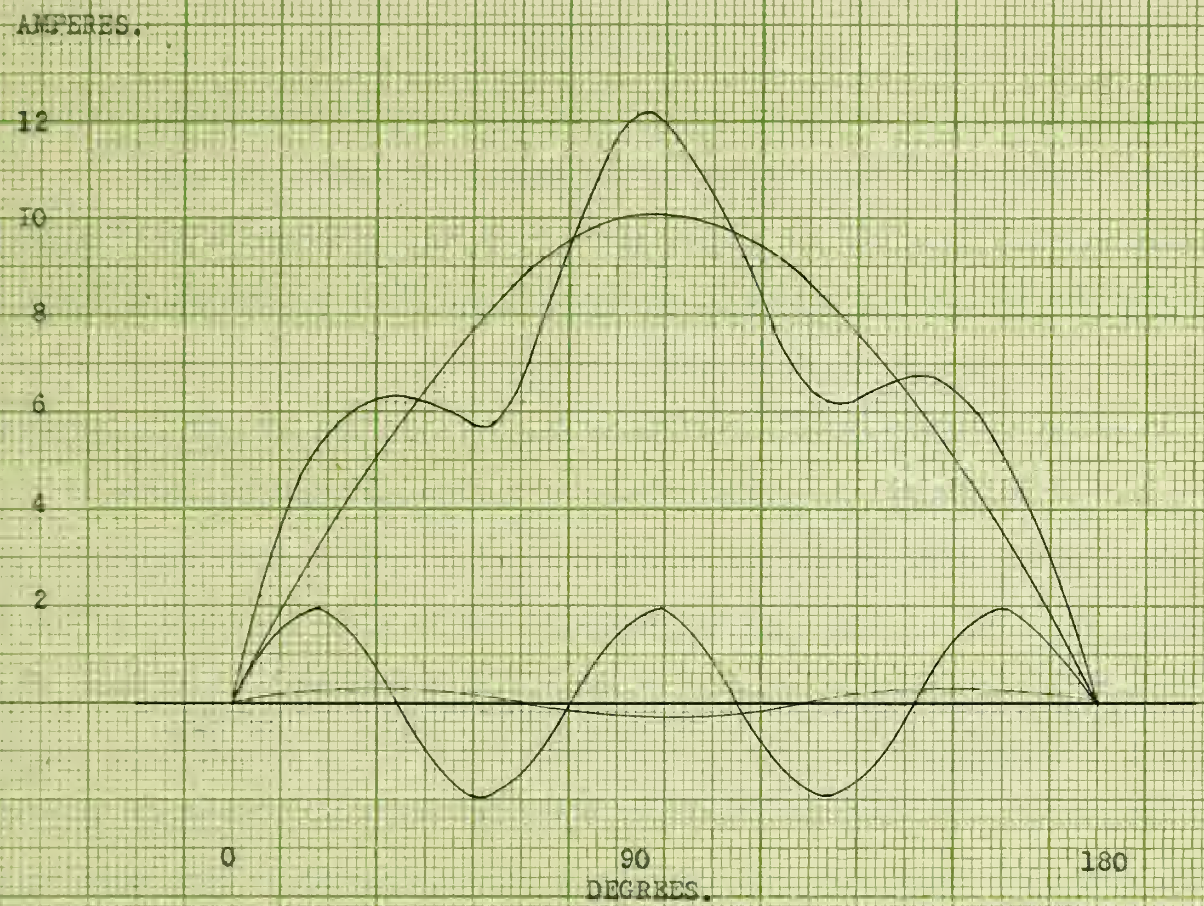

PLATE NO.O.

LINE CURRENT AND HARMONICS.

STAR PRIMARY CLOSED DELTA SECONDARY.

CONCLUSIONS.

From the study of the harmonics it is seen that the wattless or magnetizing component of the exciting current is what causes the distortion of the wave, and the higher the density at which the transformer is worked the greater the distortion. The maximum value of the exciting current depends upon the position of the harmonics relative to the fundamental. It was found that the magnitude of the triple harmonic in the delta secondary connection of three transformers was dependent upon the kind of connection made in the primaries, that is, whether star or delta. The magnitude of the effective value of the triple in the case of the star primary was much greater than in the case of the delta primary because there was no path for the triple in the star primary and it had to flow in the delta secondary while in the delta primary connection and delta secondary there are two paths for the triple. It was also shown that no matter what the distortion of a wave shape was, when caused by the same harmonics, the effective value of the wave remains the same.

From thestudy of harmonics it would ap ear that in the case of voltages higher harmonics would play an important part in the design of transformers as to insulation. For in the case of large transformers a peaked wave of electromotive force would have a much greater maximum value than effective value and in many cases these peaks could do much damage to insulation. It would appear that the quality of the insulation would thus be governed to a great extent by the shape of the wave or in other words by the positions of the harmonics relative to the fundamental and a sine wave of electromotive force would be desired or a wave similar to a sine so as not to have the high peaks. Thus while harmonics causing peaked waves in the exciting current are of no harm except for a little increase in heating, harmonics

in voltage waves have an important part in the design and life of transformers .